Minitest 4 Review
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1. Calculate $\int (x(x+2)^2 + \sec^2 x)\,dx$.

   Ans: $\frac{1}{4}x^4 + \frac{4}{3}x^3 + 2x^2 + \tan x + C$

2. (a) Using <u>midpoints</u> as your evaluation points, find an approximation for $\int_3^5 (2x-7)dx$ using 4 rectangles.

   Ans: 2

   (b) Use formula(s) from geometry to calculate the exact value of $\int_3^5 (2x-7)dx$.

   Ans: 2

   (c) Use the Fundamental Theorem of Calculus (the Evaluation Theorem) to evaluate $\int_3^5 (2x-7)dx$. How close is your answer to 2(a)? Explain.

   Ans: 2. This is identical to what we got in parts (a) and (b) because the approximation using the midpoint gives you the exact answer, since the over and under approximation cancel each other out.

3. Calculate the definite integrals:

   (a) $\int \left(\frac{x^3 - x}{x^4} \right) dx$

   Ans: $\ln |x| + \dfrac{1}{2x^2} + C$

   (b) $\int_0^1 \left(\frac{1}{1+x^2} \right) dx$

   Ans: $\pi/4$

   (c) $\int_0^{\pi^2} \frac{\sin \sqrt{x}}{\sqrt{x}}\,dx$

   Ans: 4

4. Find $h'(x)$ if $h(x) = \int_0^{e^{2x}} \sin^3 t\,dt$.

   Ans: $2e^{2x} \sin^3(e^{2x})$

5. Suppose a particle on a line has velocity $v(t) = t(2-t)$, for t-values between 0 and 4.

   (a) Explain in words the difference between the *displacement* of the particle and the *total distance traveled* from $t = 0$ to $t = 4$.

   Ans: The displacement is the difference in location from where the particle started to where it ended, you find it by calculating $\int_0^4 2(2-t)dt$. The total distance traveled is when you calculate both the forward and backward distances as positive. You find this by calculating

   $$|\int_0^2 2(2-t)dt| + |\int_2^4 2(2-t)dt|$$

(b) Now calculate each of them using integrals.

Ans: The displacement is equal to -16/3, the total distance is equal to 8.